Zoom Class 27/04/2020.

Chord of Curvature parallel to x-axis

$$= 2\rho \sin\psi$$

Chord of Curvature parallel to y-axis

$$= 2\rho\cos\psi$$

$\sin\psi$ and $\cos\psi$ are calculated by the relation $\frac{dy}{dx} = \tan\psi$

Chord of curvature passing through the origin

$$= 2\rho \sin\phi$$

where $r\frac{d\theta}{dr} = \tan\phi$

Ques Prove that the chord of curvature parallel to the axis of y for the curve $y = a \log \sec\frac{x}{a}$ is of constant length.

Soln $\because$ Chord of curvature parallel to y-axis $= 2\rho\cos\psi$

To find 'ρ' we have

$$y = a \log\sec(x/a) \quad \text{—— (1)}$$

$$\Rightarrow \frac{dy}{dx} = \cancel{a}\cdot\frac{1}{\sec(x/a)}\cdot\sec(x/a)\tan(x/a)\cdot\frac{1}{\cancel{a}}$$

$$\Rightarrow \frac{dy}{dx} = \tan\left(\frac{x}{a}\right) \quad \text{---} (2)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{a}\sec^2\left(\frac{x}{a}\right) \quad \text{---} (3)$$

$$\therefore \rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{\{1+\tan^2(x/a)\}^{3/2}}{1/a\,(\sec^2(x/a))}$$

$$= \frac{a\{\sec^2(x/a)\}^{3/2}}{\sec^2(x/a)} = a\sec^2(x/a).$$

Also $\tan\psi = \frac{dy}{dx}$

$$\Rightarrow \tan\left(\frac{x}{a}\right) = \tan\psi$$

$$\Rightarrow (x/a) = \psi$$

$$\Rightarrow \cos\psi = \cos\left(\frac{x}{a}\right)$$

$\therefore$ Chord of curvature parallel to y-axis

$$= 2\rho\cos\psi = 2a\sec\left(\frac{x}{a}\right)\cos\left(\frac{x}{a}\right)$$

$$= 2a = \text{constant}.$$

Proved

Ques Prove that the chord of curvature through the pole of the curve

$$p = f(r) \text{ is } \frac{2f(r)}{f'(r)}$$

Soln $\because$ $\rho = r\dfrac{dr}{dp}$ and $p = r\sin\phi \Rightarrow \sin\phi = \dfrac{p}{r}$

and chord of curvature through pole is

$$2\rho\sin\phi = 2\cdot \cancel{r}\frac{dr}{dp}\cdot\frac{p}{\cancel{r}}$$

$$= 2p\cdot\frac{dr}{dp} = 2p \Big/ \frac{dp}{dr}$$

$$= \frac{2f(r)}{f'(r)}$$

Proved

Ques Find the chord of curvature through the pole of the curve $r = a(1+\cos\theta)$

Soln Given curve is $r = a(1+\cos\theta)$ ——— (1)

$\Rightarrow r_1 = a(-\sin\theta) = -a\sin\theta$ ——— (2)

and $r_2 = -a\cos\theta$ ——— (3)

We know that chord of curvature at pole is $2\rho\sin\phi$ ——— (4)

Now, to find ρ we use eqn (1), (2) and (3)

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - r r_2}$$

$$= \frac{\left[a^2(1+\cos\theta)^2 + a^2\sin^2\theta\right]^{3/2}}{a^2(1+\cos\theta)^2 + 2a^2\sin^2\theta + a^2(1+\cos\theta)\cos\theta}$$

$$= \frac{a^3\{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta\}^{3/2}}{a^2\{1 + 2\cos\theta + \cos^2\theta + 2\sin^2\theta + \cos\theta + \cos^2\theta\}}$$

$$= \frac{a\{2 + 2\cos\theta\}^{3/2}}{\{1 + 2(\cos^2\theta + \sin^2\theta) + 3\cos\theta\}} = \frac{2^{3/2}a\{1+\cos\theta\}^{3/2}}{\{3 + 3\cos\theta\}}$$

$$= \frac{2^{3/2}a}{3}\frac{\{1+\cos\theta\}^{3/2}}{(1+\cos\theta)} = \frac{2^{3/2}a}{3}\{1+\cos\theta\}^{1/2}$$

$$= \frac{2^{3/2}a}{3}\{2\cos^2\theta/2\}^{1/2} = \cancel{2^{3/2}2^{1/2}\cos^2\theta/2}$$

$$= \frac{2^{3/2}\cdot 2^{1/2}\cdot a\{\cos^2(\theta/2)\}^{1/2}}{3} = \frac{2^2}{3}a\{\cos(\theta/2)\}^{1/2\cdot 2} \quad \text{— (5)}$$

$$= \frac{4a}{3}\cos\left(\frac{\theta}{2}\right)$$

Also $\tan\phi = r\dfrac{d\theta}{dr} = r\cdot\dfrac{1}{r_1} = \dfrac{a(1+\cos\theta)}{-a\sin\theta}$

$$\Rightarrow \tan\phi = \frac{2\cos^2(\theta/2)}{-2\sin(\theta/2)\cos(\theta/2)}$$

$$\Rightarrow \tan\phi = -\cot\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \tan\phi = +\tan\left\{\frac{\pi}{2} + \frac{\theta}{2}\right\} \Rightarrow \phi = \frac{\pi}{2} + \frac{\theta}{2} \quad \text{— (6)}$$

Using (5), (6) in (4) we get

$$2\rho\sin\phi = 2\cdot\frac{4a}{3}\cos(\theta/2)\cdot\sin\left(\frac{\pi}{2} + \frac{\theta}{2}\right)$$

$$\Rightarrow 2p\sin\phi = \frac{8a}{3}\cos\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$$

$$= \frac{8a}{3}\cos^2\left(\frac{\theta}{2}\right)$$

$$= \frac{8a}{3}\left\{\frac{1+\cos\theta}{2}\right\} = \frac{4a}{3}\{1+\cos\theta)$$

$$= \frac{4}{3}\cancel{a}\cdot\frac{r}{\cancel{a}} = \frac{4}{3}r$$

Ans